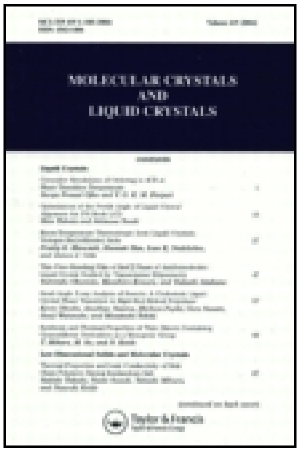

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Abstract Surface stabilized smectic liquid crystal devices can be used in a wide variety of optical filters for providing very rapid discrete, and continuous tunability. One such example, analyzed here, is the discrete switching Ferroelectric Liquid Crystal Polarization Interference Filter (FLCPIF). This filter is implemented using binary swiching surface stabilized FLC's and linear polarizers. A single-stage FLCPIF is analyzed, and the number of output spectra is determined which optimizes performance and makes the most efficient use of the devices. The extension of principles to multiple-stage FLC-PIF's, and FLCPIF's which include passive retarders is discussed.

INTRODUCTION

There are many applications that would currently benefit from the availibility of inexpensive, reliable, rapidly tunable optical filters. These include high resolution applications, such as Wavelength Division Multiplexng (WDM) for optical communications,[1] moderate to high resolution applications, such as remote sensing and spectrometry,[2] and relatively low resolution applications, such as colorimetry and color display.[3] In general, it is desirable to have the capability of continuous, random access tunability of the filter transmission peak throughout the spectral band of interest. However, in many instances it is not necessary to address all wavelengths within a spectral band. Furthermore, many applications require detection of optical power at only a handful of fixed optical bands. These include detection of radiation at specific optical resonance or transition wavelengths, differential absorption detection to measure species concentrations, and color filtering, where arbitrary colors can be visually perceived by rapid modulation between three primary bands. Such cases are well suited for FLCPIF implementations, in which the filter design and driving scheme are quite simple. For low resolution applications, all-FLCPIF's can be used to generate electronically selectable spectra, while moderate/high resolution applications require the addition of fixed retarders. In this paper, the switching rules for a single-stage all-FLCPIF, and an FLCPIF stage which includes a passive retarder are presented. The extension of the FLCPIF tuning principles to multiple stage designs is also considered.

It is well known that thin layers of Smectic C* liquid crystals (typically a few microns), experience strong surface interactions with the bounding substrates. Surface interactions in thin cells produce complete suppression of the helix formation, resulting in two stable director orientations[4]. The switching of an electric field applied normal to the substrates, produces a ferroelectric torque which reorients the molecular directors. The molecules can be switched rapidly (10's of microseconds) between the two states, which are separated by twice the tilt angle of the molecular director. For an ideal "bookshelf" structure, the device functions as a bistable, switchable birefringent element. That is, the FLC device functions as a waveplate with fixed retardation (determined by the layer thickness), which can be rotated about the normal to the substrates by switching the polarity of the applied electric field. Because of the high tilt of the molecules ($\pm\pi/8$), FLC's can be used in a binary pseudo-variable retarder mode. Here, one of the FLC switched states aligns the director with (or normal to) the linear input polarization, producing no change in the state of polarization, while a retarder with orientation $\pi/4$ is seen in the other state. Typically, zero-order FLC half-wave retarders are switched between crossed polarizers for binary intensity modulation. However, FLC devices can be used in a similar configuration to generate or shift spectra in PIF's.

The simplest Polarization Interference Filter (PIF) requires three components: an entrance polarizer, an exit polarizer (or polarization analyzer), and a single birefringent element (bounded by the polarizers). This sequence of elements, referred to here as a filter stage, is the simplest stand-alone PIF. By this definition, the Lyot filter,[5] for example, is a cascading of filter stages, while the Solc filter[6] is intrinsically a single-stage filter. A multiple stage filtering scheme typically involves selection of specific relative thicknesses of birefringent elements in each stage in order to produce a desired output spectrum. A Lyot filter, for example, achieves optimum finesse, with the geometric series of birefringent element thickness ratios; (1:2:4....). The finesse, given as the number of pass-band full-widths in a period of the filter spectrum (or Free-Spectral-Range (FSR)), scales as 2^M for an M-stage (and therefore M birefringent element) Lyot filter. By contrast, the Solc filter finesse scales as M, thus making inefficient use of the birefringent elements. It will be later shown that the optimum arrangement of elements in a FLCPIF is similar to a single stage of a Lyot filter. Due to the independence of filter stages, the overall transmission function from a multiple stage filter is given by the product of the transmission functions of the individual stages. With this in mind, a great deal of information can be obtained about the overall performance of a multiple stage FLCPIF, by examining the performance of a single filter stage.

ANALYSIS OF SINGLE-STAGE FLCPIF

A general single-stage FLCPIF consists of a series of FLC active birefringent elements, bounded by input and exit polarizers. One or more passive birefringent elements may also be included in the stage. In the analysis, the FLC's will be treated as uniaxial crystals, with optic axis (directors) contained in the plane of the substrates. It will be assumed that the devices exhibit a uniform, planar director distribution. The FLC's have fixed retardation given by

$(\Gamma_1, \Gamma_2, \ldots \Gamma_N)$, where N is the total number of devices, oriented such that the optic axes of the FLC's form angles $(\theta_1, \theta_2, \cdots \theta_N)$ with respect to the input polarization. The exit polarizer is oriented at arbitrary angle θ_o. For a filter stage which contains N active FLC devices, each of which exhibits two stable states, there are in general 2^N unique output spectra, which are electronically selectable. Here, the angle of the molecular director of each FLC can be modulated between angles $(\theta_1, \theta_1 + 2\phi)$, $(\theta_2, \theta_2 + 2\phi)$.... $(\theta_N, \theta_N + 2\phi)$, where, ϕ is the tilt angle that the molecular director forms with the layer normal. For simplicity, the devices are assumed to have equal tilt angles. In general, each of the 2^N switched states results in a different output spectrum, or filter transmission function. However, some of these outputs may not be suitable for optical filtering applications. To be "useful", an output must; (1) be unique, and; (2) satisfy performance specifications for the particular application. While uniqueness is easily defined, utility of a particular transmission function is application specific.

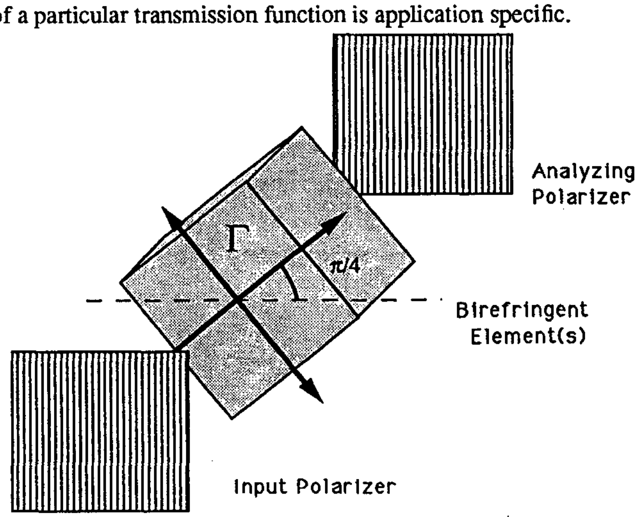

FIGURE 1 Single-stage FLCPIF showing optimum orientation of polarizers and waveplate(s).

While there are 2^N outputs of a general FLCPIF stage, only specific orientations of polarizers and waveplates generate spectra which are considered ideal for filtering applications. Typically, the requirement in optical filtering is for maximum transmission of a particular band of wavelengths (ideally unity), with strong blocking (ideally a null in transmission) of all wavelengths outside of this band. It is desirable to achieve this filtering operation with a minimum number of optical components. The requirements for maximum spectral discrimination with efficent use of components determines specific relative orientations for all components in the stage. Figure 1 shows the optimum orientation of elements in a single-stage filter. Here, light passing first through a linear polarizer is incident on a single birefringent element oriented at angle $\pi/4$ with respect to the input polarization. The function of the retarder is to produce two orthogonally polarized waves from a single (linearly polarized) input wave, and to introduce a wavelength dependent phase shift between the waves. The phase shift is a consequence of the difference in optical path-length experienced between ordinary and extraordinary waves. The amplitudes of the two waves are determined by the orientation of the optical field vector with respect to the crystal axes. For a slab of positive uniaxial material ($n_e > n_o$), with birefringence $\Delta n = (n_e - n_o)$, of thickness d, the two waves exit the crystal with a phase difference of,

$$\Gamma(\lambda) = 2\pi \frac{\Delta n(\lambda) d}{\lambda}, \tag{1}$$

where, λ is the free space wavelength of incident radiation. Due to dispersion, the birefringence, Δn, is a function of wavelength. This phase delay represents a wavelength dependent change in the state of polarization. Light exiting a waveplate oriented at $\pi/4$ with respect to the input polarization is elliptically polarized with major axis parallel (or perpendicular) to the input polarization. Assuming an orientation of $\pi/4$, the Jones vector[7] for light exiting the waveplate, $\mathbf{E}'$, is given by,

$$\mathbf{E}' = E \begin{bmatrix} \cos[\Gamma(\lambda)/2] \\ i \sin[\Gamma(\lambda)/2] \end{bmatrix}, \tag{2}$$

where, E is the amplitude of the linearly polarized optical field incident on the birefringent element. The wavelength dependent ellipticity can be analyzed by interference at the exit polarizer, producing a wavelength dependent optical transmission. The transmission spectrum resulting from this single-stage, using parallel or crossed analyzing polarizers, is given by,

$$T(\lambda) = \begin{cases} \cos^2\left[\dfrac{\pi (m+1) \lambda_o}{\lambda}\right] & \textit{PARALLEL} \\ \sin^2\left[\dfrac{\pi (m+1/2) \lambda_o}{\lambda}\right] & \textit{CROSSED} \end{cases} \tag{3}$$

where the waveplate is an m-order (m=0,1,..) full-wave retarder at design wavelength λ_o (with parallel polarizers), or an m-order half-wave retarder at λ_o (with crossed polarizers). This corresponds to the $(m+1)^{th}$ maximum in transmission at λ_o. The requirement for orientation $\pi/4$ can now clearly be seen in the above equations, as it produces two equal amplitude waves, which generates pure sinusoidal transmission spectra. For parallel polarizers, interference of these waves produces theoretically 100% transmission for wavelengths which exit the birefringent element with field components in phase, and theoretical nulls (0% transmission) for wavelengths which exit the birefringent element with field components out of phase. The converse is true for the case of crossed polarizers. A figure of merit which indicates the effectiveness of a filter stage in spectral descrimination is the Spectral Contrast Ratio (SCR). This is given by the ratio of the mean transmission at peak wavelengths to the mean transmission at null wavelengths, over the spectral band in which the filter stage must function. Using the ideal transmission functions of Equation 3, the SCR tends toward infinity. In practice, stray light from non-ideal polarizers and scattering from defects in the birefringent elements typically limits the SCR. Using active FLC elements, a slightly non-uniform director distribution and scattering from defects usually limits the SCR in FLC's to a value < 10,000, depending upon device thickness. However, this condition represents the ideal situation, or pure spectrum, in which the SCR is limited only by component nonidealities. In designing an FLCPIF with multiple devices, it is the polarization effects associated with the switching scheme, which limits the number of pure outputs to some fraction of the 2^N available. This can best be illustrated by considering a specific example of an all-FLC filter stage. The filter stage analyzed in this paper, as illustrated in Figure 2, contains three active FLC elements. This number is significant because it is the minimum number of elements required to generate all distinct polarization states of significance, as will become apparent. The results of this analysis can be generalized to the case of an N FLC filter. In addition, this example will indicate the requirements on device fabrication for uniqueness of spectral output.

All-FLCPIF Stage

The filter stage considered here is illustrated in Figure 2. Each of the active waveplates can be independently switched between director orientations of 0 or $\pi/4$, with respect to the input polarization.

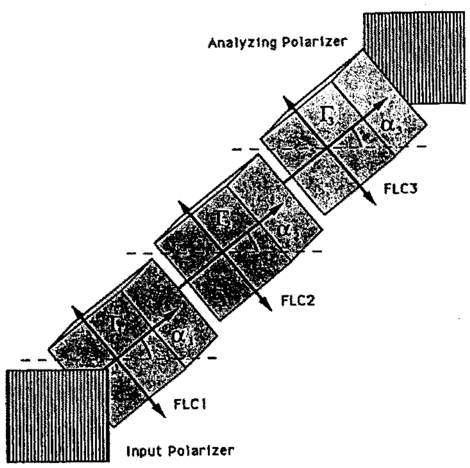

FIGURE 2 Single-stage all-FLCPIF, using three surface stabilized Smectic C* devices.

Because each of the FLC devices switches between two stable states, a particular switched state of the filter can be represented by a binary number. A state of (011), for example, corresponds to $\alpha_1 = 0$, $\alpha_2 = \pi/4$, and, $\alpha_3 = \pi/4$. In order to determine each output of the filter stage, Jones calculus is employed. The optical field transmitted by the filter is given by the matrix equation,

$$E'(\Gamma,\alpha) = P_x W_N(\Gamma_N,\alpha_N) \ldots\ldots W_2(\Gamma_2,\alpha_2) W_1(\Gamma_1,\alpha_1) P_x E, \quad (4)$$

where, E and E' are the column matrices giving the incident and transmitted optical fields, respectively. Also, Γ and α are the vectors giving the retardation and orientation of the waveplates, respectively,

$$\Gamma = (\Gamma_1, \Gamma_2, \ldots \Gamma_N), \quad \alpha = (\alpha_1, \alpha_2, \ldots \alpha_N). \tag{5a,b}$$

The matrix, $\mathbf{P_x}$, corresponds to an x-oriented polarizer, given by,

$$\mathbf{P_x} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} . \tag{6}$$

The general matrix for a waveplate, with retardation Γ_i, and orientation α_i, is given by,

$$\mathbf{W}(\Gamma_i, \alpha_i) = \begin{bmatrix} \cos[\Gamma_i(\lambda)/2] - i\cos[2\alpha_i]\sin[\Gamma_i(\lambda)/2] & -i\sin[2\alpha_i]\sin[\Gamma_i(\lambda)/2] \\ -i\sin[2\alpha_i]\sin[\Gamma_i(\lambda)/2] & \cos[\Gamma_i(\lambda)/2] + i\cos[2\alpha_i]\sin[\Gamma_i(\lambda)/2] \end{bmatrix} . \tag{7}$$

This matrix reduces to two matrices representing the two director orientations corresponding to unswitched ($\alpha_i = 0$) and switched states ($\alpha_i = \pi/4$), respectively,

$$\mathbf{W}(\Gamma_i, 0) = \begin{bmatrix} e^{-i\Gamma_i(\lambda)/2} & 0 \\ 0 & e^{i\Gamma_i(\lambda)/2} \end{bmatrix} , \quad \mathbf{W}(\Gamma_i, \pi/4) = \begin{bmatrix} \cos[\Gamma_i(\lambda)/2] & -i\sin[\Gamma_i(\lambda)/2] \\ -i\sin[\Gamma_i(\lambda)/2] & \cos[\Gamma_i(\lambda)/2] \end{bmatrix} . \tag{8a,b}$$

Note that absolute phase factors are neglected in these Jones matrices, as only relative phase is of consequence when generating an intensity transmission function. A generalized output transmission function could be derived by substituting the matrices of Equations 6 and 7 into Equation 4, giving an expression for the output field, which includes the effect of N retarders with abitrary retardation and orientation. However, due to the binary nature of the devices, it is algebraically simpler to generate transmission functions for each specific switched state of the stage, using Equations 8a, and b. The transmission function for each of the 2^N (=8) outputs of the three-FLC stage, given by $T(\Gamma,\alpha) = |E'/E|^2$, can be expressed in the following form, along with the corresponding binary switching representation,

$$T(\alpha,\Gamma) = 1 \qquad (000)$$

$$\cos^2[\Gamma_1/2] \qquad (100)$$

$$\cos^2[\Gamma_2/2] \qquad (010)$$

$$\cos^2[\Gamma_3/2] \qquad (001)$$

$$\cos^2\left[\frac{\Gamma_1+\Gamma_2}{2}\right] \qquad (110) \qquad\qquad (9)$$

$$\cos^2\left[\frac{\Gamma_2+\Gamma_3}{2}\right] \qquad (011)$$

$$\cos^2\left[\frac{\Gamma_1+\Gamma_3}{2}\right] + \sin^2\Gamma_2/2\,\sin\Gamma_1\,\sin\Gamma_3 \qquad (101)$$

$$\cos^2\left[\frac{\Gamma_1+\Gamma_2+\Gamma_3}{2}\right] \qquad (111)$$

For the three FLC stage with parallel polarizers, the 8 outputs consist of the source spectrum, 6 pure spectra, and 1 non-pure spectrum. Switching all of the FLC's (111) produces a pure spectum determined by the sum of the waveplate retardations. One or more unswitched FLC's preceeding one or more switched FLC's (001,011) generates pure spectra, as the unswitched FLC's do not change the state of the input polarization. One or more switched FLC's preceeding one or more unswitched FLC's (100,110) also generates pure spectra. This is not so obvious, as the light entering the unswitched FLC('s) is in general elliptically polarized. Consequently, there is a change in the state of polarization induced by the unswitched device(s). However, because the unswitched FLC('s) are oriented along the exit polarizer, this change in polarization represents an exchange between elliptical and $\pm\pi/4$ oriented linear polarization, determined by the retardation. Therefore, the S_1 component of the Stokes vector,[8] which determines the output transmission, is unchanged. The state (010) is a combination of both of these cases, and is therefore also pure. The remaining state corresponds to an unswitched FLC bounded by switched FLC's (101). Here, the change in state of polarization induced by the unswitched FLC, preceeding the remaining switched FLC, produces a term which is additive with the pure transmission term. In general, this additive term produces a SCR which is limited by polarization effects. Consequently, this output is not useful for applications requiring a high degree of spectral discrimination.

Generalization to N-FLCPIF Stage

The various output states, determined by the analysis of the 3-FLCPIF, can be generalized to a filter stage containing N FLC's. In general, one or more unswitched FLC's bounded by one or more switched FLC's produces a non-pure output spectrum. This additive transmission term results in a polarization limited SCR. In a stage containing several FLC's, this non-ideal (101) sequence may occur in several of the 2^N states, often in more than one position in the stage. The binary contracted representation for switched states that produce pure spectra are, independent of the number N of FLC's, (1), (01), (10), and, (010). Again, the output (0) results in full transmission of the source spectrum for parallel polarizers, and a null in transmission for crossed polarizers. Though this does not represent a filtering operation, it is necessary in certain applications to have either a dark state, or one in which the source spectrum can be analyzed. The number of occurrences of the four sequences which yield pure spectra gives the number of useful filtering operations. The remaining terms contain one or more occurrences of the sequence (101).

The transmission function for a particular output state, generalized to the case of an N-FLCPIF stage, can be written in a separable form as,

$$T(\Gamma,\alpha) = T_P(\Gamma,\alpha) + T_E(\Gamma,\alpha). \tag{10}$$

Here, $T_P(\Gamma,\alpha)$, represents the desired pure spectrum, given by,

$$T_P(\Gamma,\alpha) = \cos^2\left[\frac{1}{2}\sum_{i=1}^{N}\Gamma_i\ \delta(\alpha_i - \pi/4)\right], \tag{11}$$

and, $T_E(\Gamma,\alpha)$ is a non-ideal additive term, the latter being non-zero for one or more occurrances of the (101) sequence. The additive term, $T_E(\Gamma,\alpha)$, cannot be generalized to the N FLC stage, as the (101) sequence may occur in several places in the stage for a particular state. The number of pure output spectra, N_p, from a stage containing only N FLC's, is determined to be,

$$N_P = \sum_{n=1}^{N} n = \frac{N\,(N+1)}{2}. \tag{12}$$

This can of course be represented as a fraction of the $(2^N - 1)$ spectra produced by the stage. For the case $N = 2$, 100% of the states are pure, which is reduced to 50% for $N = 5$. The case $N = 5$, however, corresponds to 15 pure output spectra.

Uniqueness of Spectral Transmission

In the previous section, a relation for the maximum number of pure spectra for an all-FLC filter stage was given. However, each of the states given in Equation 9 for the 3-FLC case do not necessarily represent unique spectra. In order to make most efficient use of the FLC devices, it is necessary that each of the $N(N+1)/2$ pure spectra also be unique. This clearly requires that the retardation of each of the FLC layers be unique, and that the various combinations of the sums (or differences, depending upon the switching scheme) of FLC retardations be unique. For example, it can be seen from Equation 9 that three FLC's with equal retardation produce a total of 3 unique output spectra. In general, an N FLCPIF stage, which uses FLC's with identical thickness, produces only N unique output spectra. Consider, for example, a geometric series of waveplate thicknesses ($\Gamma_1 = \Gamma_0, \Gamma_2 = 2\Gamma_0, \Gamma_3 = 4\Gamma_0$). This generates the ideal seven unique output spectra corresponding to a linear shift in retardation, with the spectrum resulting from the $5\Gamma_0$ state being non-pure. In summary, maximizing the number of unique output spectra requires that individual FLC retardations and all combinations of sums of retardations be unique.

FLCPIF With Passive Retarder

In many instances it is necessary to insert a passive birefringent element into the FLCPIF, optimally oriented at $\pi/4$. The addition of one (or more) passive waveplates into the stage occurs for two reasons: (1) First, the passive waveplate allows generation of a spectrum in the (0) state (none of the FLC's switched), which simply increases the number of pure output spectra by one. (2) Second, and more importantly, a bias retardation is often required to produce a higher resolution than that practical with FLC's alone (due to thickness limitations and required voltages). In this instance, the FLC's are used to augment the retardation of the passive element when switched, thereby shifting the output spectrum. In this case, thick FLC cells are not required, as they are not needed to generate a high-order spectrum.

The number of pure output spectra is now determined for a single stage containing parallel (or crossed) polarizers, a single passive retarder oriented at angle $\pi/4$, and N FLC cells. As before, these devices can be switched between orientations of 0 or $\pi/4$ by reversing the sign of the applied electric field. Again, the outputs from the stage can be given in a binary representation. Naturally, a passive waveplate is always given by a (1) in the binary switching representation. The number of pure output spectra for a particular N-FLC filter stage is a function of the position of the passive waveplate in the stage. This can be shown by counting all binary switched states which do not contain the (101) sequence, for each position of the passive waveplate. Using $N = 5$, for example, it can be shown that 6 pure output spectra result with the passive element in first or last position (order of appearance of the waveplate with respect to the input polarizer). However, by placing the passive element in either the third or fourth position, twelve pure output spectra result. For the number of FLC's ($N = 1,2,3,4,5$), the results of this analysis indicate that, with the passive element placed in the optimum position, 2,4,6,8 and 12 pure spectra result, respectively. Thus, for $N < 3$, more pure spectra result with the presence of a passive retarder, while for $N > 3$ more pure spectra are produced by an all-FLC filter.

MULTIPLE-STAGE FLCPIF

Usually, applications require higher selectivity than that provided by a pure sinusoidal transmission function. Consequently, it is necessary to cascade filter stages to increase the finesse. In an actual implementation, a specific design may include passive PIF stages (or any passive filter), and PIF stages with one or more FLC devices. As mentioned previously, the output of a multiple-stage PIF is given by the product of the transmission spectra of the individual stages. The increase in finesse is therefore accomplished by designing filter stages which produce a coincident transmission maximum (the desired output center wavelength), with different (complementary) blocking characteristics. In designing a multiple-stage FLCPIF, there is the additional design complexity of generating such transmission spectra for each output band, using a minimum number of active devices. Naturally, the design depends strongly upon the nature of the source being filtered, and the performance specifications mandated by the particular application. For this reason, it becomes difficult to quantify what a "good" transmission function is in a multiple stage implementation. Often, performance of a multiple stage filter is determined by a Spectral-Signal- to-Noise-Ratio (SSNR) measurement. This is given as the ratio of the spectrally integrated transmission in the full-width of the information band, to the integrated transmission in the band over which the filter must block. A more accurate SSNR can be obtained by weighting the transmission function by the specific source power spectrum, if known. While SSNR constraints vary, depending upon the application, it is usually desirable to optimize this parameter, (as spectral purity of each output is usually the desired goal) while minimizing the number of FLC devices. Such useful spectra are generated most effectively by FLCPIF stages producing the pure spectra described in this paper. Beyond this, multiple-stage FLCPIF design usually becomes a problem best approached with computer modelling.

CONCLUSIONS

In summary, an active Ferroelectric Liquid Crystal Polarization Interference Filter (FLCPIF) has been analyzed. The switching scheme, and the number of pure output spectra from a single-stage all-FLCPIF, and a single-stage PIF with a passive birefringent element were determined. The condition for uniqueness of output transmission function was discussed, as well as the extension of filter designs to multiple-stage implementations. The FLCPIF is an effective and practical method for rapid switching between a moderate number (order 10) of output spectral bands, centered at specific wavelengths. The FLCPIF offers excellent reproducability, simple digital driving schemes, and low power consumption, due to the bistable material response.

REFERENCES

1. C. Lin, Optoelectronic Technology and Lightwave Communications Systems (Van Nos trand Reinhold, New York, 1989).
2. Title, A.M., and W.J. Rosenberg, Opt. Eng., 20, 815, (1981).
3. White, J.C., Phys. Tech., 19, 912 (1988).
4. Clark, N.A. and S.T. Lagerwall, Appl. Phys. Lett., 36, 899, (1980).
5. Lyot, B., Ann. Astrophys., 7, 31, (1944).
6. Solc, I., JOSA, 55, 621, (1965).
7. Jones, R.C., JOSA, 31, 488 (1941).
8. Born, M., and E. Wolf, Principles of Optics (Pergamon Press, New York, 1980), p.30.